

# SPECTRAL ANALYSIS OF EEG SIGNALS BY USING WAVELET AND HARMONIC TRANSFORMS

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In this study, wavelet transforms and FFT methods, which transform method is better for spectral analysis of the brain signals are investigated. Statistical and Fourier methods are traditional techniques and tools to analyze time series signals in general, including biomedical data. In this paper as spectral analysis tools, wavelet transform and harmonic transform are used. Both transform methods are applied to electroencephalogram (EEG) of a possibly epilepsy patient and are compared. For this purpose in the harmonic transform case, the variations of first-sixth-order harmonic amplitudes and phases provide a useful tool of understanding the large- and local-scale effects on the parameters. Moreover, temporal and frequency variations of variables are also detected by wavelet transforms. The results of this study are compared with previous studies. The comparison of results show that the wavelet transform method has more advantage in detecting brain diseases.

**Keywords:** *Wavelet transforms, FFT, EEG, Epilepsy.*

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## 1. INTRODUCTION

Brain is the most important organ which controls the functioning of the human body including heart beat and respiration. It is the portion of the vertebrate central nervous system that is enclosed within the cranium, continuous with the spinal cord, and composed of gray matter and white matter. It also is the primary center for the regulation and control of bodily activities, receiving and interpreting sensory impulses, and transmitting information to the muscles and body organs.

Seizures are sudden surge of electrical activity in the brain that usually effects how a person feels or acts for short duration times. Epilepsy is a common neurological disorder characterized by recurrent seizures. There are more than 40 types of epilepsy which can be characterized by their different energy distribution in different levels of decomposition using wavelet transform. About 1% of the world populatin is suffering from epilepsy and 30% of epileptic patients are not cured by medication and may need surgery.

Electroencephalography (EEG) is a record of electrical activity along the scalp produced by the firing of neurons within the brain. Recorded EEG provides graphical exhibition of the spatial distribution of the changing voltage field. A routine clinical EEG recording typically lasts 20–40 minutes (plus preparation time) and usually involves recording from multiple electrodes placed on the scalp. A routine clinical EEG recording typically lasts 20–30 minutes (plus preparation time) and usually involves recording from scalp electrodes. Routine EEG is typically used in the following clinical circumstances:

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- to distinguish epileptic seizures from other types of spells, such as psychogenic non-epileptic
- to serve as an adjunct test of brain death,
- to prognosticate, in certain instances, in patients with coma,
- to determine whether to wean anti-epileptic medications.

Fourier transform (FT) has been the traditional method applied to time series signals. On the other hand the wavelet transform has become a useful

computational tool for a variety of signal and image processing applications over the last twenty years. In this paper, the Fourier transform and wavelet transform are applied to the EEG signals obtained from the east Marmara region of Turkey and compared to each other. Also the performance of wavelet transforms and Fourier methods for spectral analysis of the brain signals are investigated. Meyer and Daubechies-4 wavelet transforms have been used to analyse the data. In <sup>6</sup>Akin applied Short-Time Fourier Transformation and wavelet transformation methods to epileptic and non-epileptic cases basically. Here we have presented scalograms for the wavelet analysis cases to show the distinct superiority of wavelets to Fourier transformation on the EEG signal case.

The paper is organized as follows: In the introduction we give a brief introduction to brain, epilepsy and EEG signals. In the second section, information regarding methodology used in this study is discussed. To be more clear, analysis methods for EEG signals are introduced. In the third section, we give information about the data used in this study. In the fourth section, we give case studies. Finally results of the applications of Fourier and wavelet methods are commented.

## 2. METHODOLOGY

The basic approach to signal analysis is to get proper information from the signal by applying the best suitable method. The conditions of reversibility of the applied method and representing the original signal by the converted form should be satisfied one by one <sup>6,7</sup> There are several methods for the spectral analysis of the brain signals. Fourier transformation has been the traditional analysis method for analyzing signals. However brain signals are non-stationary and Fourier transformation is used mainly for stationary signals. To overcome this difficulty some other analysis methods have been developed. Wavelet transformation method can overcome this difficulty and give much improved

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<sup>6</sup> A.H.Siddiqi, A. Chandio, V.Singh Bhadouria, 'Analysis and prediction of energy distribution in electroencephalogram (EEG) using wavelet transform'

<sup>7</sup> E Magosso, M Ursino, A Zaniboni and E Gardella, A wavelet-based energetic approach for the analysis of biomedical signals: Application to the electroencephalogram and electro-oculogram, Applied Mathematics and Computation Volume 207, Issue 1, 1 January 2009, Pages 42-62

results especially in the EEG signal case. The methods used in this study are Fourier and wavelet transforms. We now briefly introduce each technique.

In the Fourier transform case spectral analysis of a signal involves decomposition of the signal into its frequency (sinusoidal) components. In other words, the original signal can be separated into its subspectral components by using spectral analysis methods<sup>8</sup>. Among spectral analysis techniques, Fourier transform is considered to be the best transformation between time and frequency domains because of it being timeshift invariant. The Fourier transform pairs are expressed as below

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \quad (1)$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn}$$

(2)

where,  $W_N = e^{-j(2\pi/N)}$  and  $N = \text{length}[x(n)]$ .

Wavelet transform method splits up the signal into a bunch of signals. It can be considered as a mathematical microscope. In the wavelet method, the same signal which corresponds to different frequency bands is represented  $[2^9, 3^{10}, 4^{11}, 5^{12}, 6^{13}]$ . It only provides what frequency bands exists at what time intervals. It is developed to overcome the shortcomings of Fourier transform.

<sup>8</sup> M. Akin, 'Comparison of Wavelet Transform and FFT Methods in the Analysis of EEG Signals', Journal of Medical Systems, Vol. 26, No. 3, June 2002.

<sup>9</sup> E Magosso, M Ursino, A Zaniboni and E Gardella, A wavelet-based energetic approach for the analysis of biomedical signals: Application to the electroencephalogram and electro-oculogram, Applied Mathematics and Computation Volume 207, Issue 1, 1 January 2009, Pages 42-62

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<sup>11</sup> Hojjat Adeli, Ziqin Zhou, Nahid Dadmehr 'Analysis of EEG records in an epileptic patient using wavelet transform' 123, 69\_87(2003).

<sup>12</sup> O.A. Rosso, M.T. Martin, A. Figliola, K. Keller, A. Plastino, EEG analysis using wavelet-based information tools, Journal of Neuroscience Methods 153 (2006) 163–182

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The continuous wavelet transform (CWT) of a function  $f(t)$  with respect to some local base function (wavelet)  $\psi$  is defined as

$$W(a,b) = W_w f(b,a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \psi^* \left( \frac{t-b}{a} \right) dt, \quad a > 0 \quad (3)$$

where  $\psi^*$  is the complex conjugate of  $\psi$ . The parameter  $b$  and  $a$  are called as translation (shifting) and dilation parameters respectively. The wavelet behaves like a window function. At any scale  $a$ , the wavelet coefficients  $W(a,b)$  can be obtained by convolving  $f(t)$  and a dilated version of the wavelet. To be a window and to recover from its inverse wavelet transform (IWT),  $\psi(t)$  must satisfy

$$\int_{-\infty}^{\infty} \psi(t) dt = 0. \quad (4)$$

Although  $W(b,a)$  provides space-scale analysis rather than space-frequency analysis, proper scale-to-frequency transformation allows analysis that is very close to space-frequency analysis. Reducing the scale parameter ' $a$ ' reduces the support of the wavelet in space and hence covers higher frequencies and vice-versa therefore ' $\frac{1}{a}$ ' is a measure of frequency. The parameter ' $b$ ' indicated the location of the wavelet window along the space axis thus changing  $(b,a)$  enables computation of the wavelet coefficients  $W(b,a)$  on the entire frequency plane.

Scalograms are the graphical representation of the square of the wavelet coefficients for the different scales. They are isometric view of sequence of the wavelet coefficients verses wavelength. A scalogram clearly shows more details, identifies the exact location at a particular depth, and detects low frequency cyclicity of the signal. The scalogram surface highlights the location (depth) and scale (wavelength) of dominant energetic features within the signal, say of gamma rays, bulk density and neutron porosity of a well log.

The combinations of the various vectors of coefficients at different scales (wavelengths) form the scalogram. The depth with the strongest coefficient indicate the position were that the particular wavelength change is

taking place. The scalogram provides a good space-frequency representation of the signal.

## 2.1. Wavelet Spectrum

The total energy contained in a signal,  $f(t)$  is defined as

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \|f\|^2. \quad (5)$$

Two dimensional wavelet energy density functions is defined as  $E(a,b) = W(b,a)$ . It signifies the relative contribution of the energy contained at a specific scale “ $a$ ” and location “ $b$ ”. The wavelet energy density function  $E(a,b)$  can be integrated across “ $a$ ” and “ $b$ ” to recover the total energy in the signal using admissibility constant  $c_g$  as follows

$$E = \frac{1}{c_g} \int_{-\infty}^{\infty} \int_0^{\infty} |W(a,b)|^2 \frac{da}{a^2} db. \quad (6)$$

Wavelet spectrum denoted by  $E(a)$  is defined as

$$E(a) = \frac{1}{c_g} \int_{-\infty}^{\infty} W(a,b)^2 db. \quad (7)$$

The wavelet spectrum has a power law behavior  $E(a) \approx a^{-\lambda}$ . Wavelet Spectrum  $E(a)$  defines the energy of the wavelet coefficient (wavelet transform) for scale ‘ $a$ ’. Peaks in  $E(a)$  highlights the dominant energetic scales within the signal. The total energy contained in a 2D signal  $f(x, y)$  is defined as

$$E = \int_{c_1}^{d_1} \int_{c_2}^{d_2} |f(x, y)|^2 dx dy = \|f\|^2. \quad (8)$$

For discrete  $f$ , the total energy  $E$  is given as

$$E = \sum_m \sum_n |f(m, n)|^2 = \|f\|^2. \quad (9)$$

It may be noted that the wavelet transform of a given signal can be reconstructed. Furthermore, the total energy of the given signal and its wavelet transform are identical. If  $E(\text{Total})$  is considered to be the total energy of the signal then relative energy is given by

$$E_{\text{Relative}} = \frac{E(\text{Energy of the level to be considered})}{E(\text{Total})} \quad (10)$$

Scalogram is the graphical representation of the square of the wavelet coefficient versus wavelength. It clearly shows more details and direct low frequency cyclicity of the signal. It may be noted that scalogram is nothing but a 2-dimension wavelet energy density function.

## 2.2. Wavelet Cross-Correlation

Two signals are said to be correlated if they are linearly associated, in other words if their wavelength spectrum a certain scale or wavelength are linearly associated<sup>14, 15</sup>. Broadly speaking graphs of  $a$  versus  $E(a)$  for two signal are similar (increase or decrease together).

## 3. DATA

In this study we have used data available with us through Kocaeli University's Medical School. This study is a part of an on-going research project funded by Kocaeli University's Research Foundation. EEG of several possibly epilepsy patients get recorded at the hospital. In the present study 3

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patient's data have been analyzed. According to our analysis sample 1 shows no symptoms of epilepsy while sample 2 shows one and sample 3 shows multiple seizures.

#### 4. CASE STUDIES

The Fourier transforms of the EEG signals are displayed in Figures 1,2 and 3.

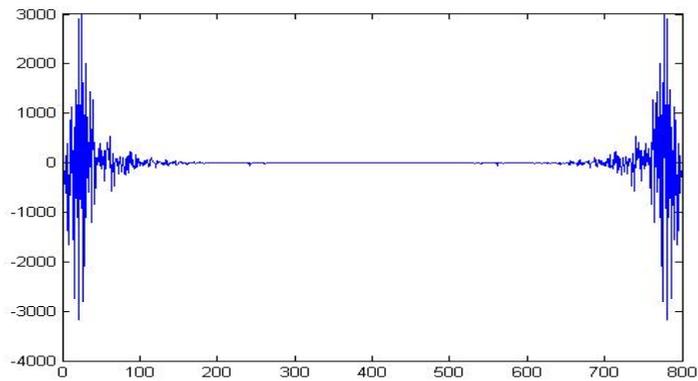


Figure 1-Fourier transformation applied to s1

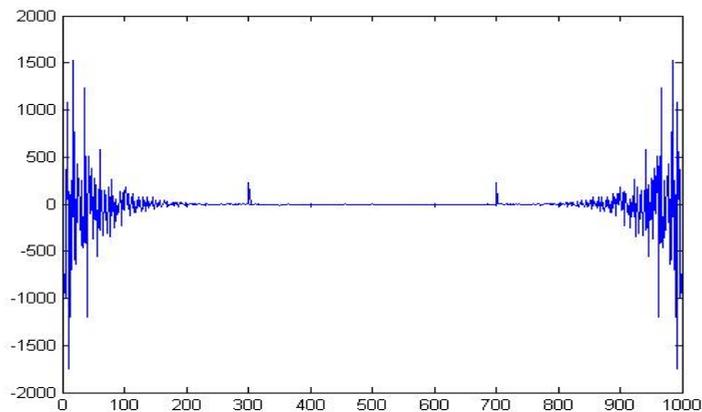


Figure 2-Fourier transformation applied to s2

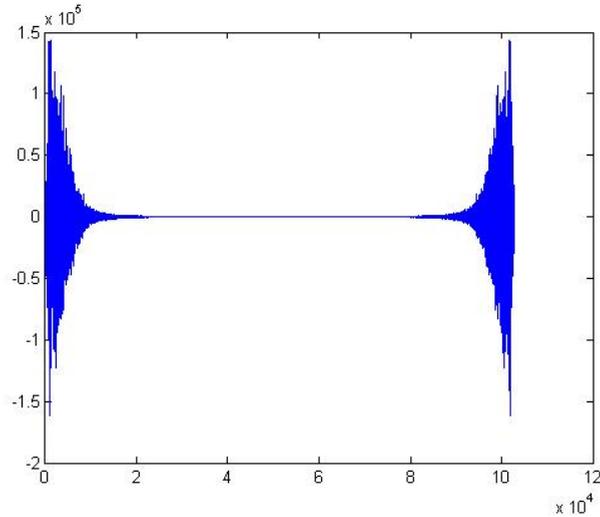


Figure 3-Fourier transformation applied to s3

In Figure 1 it can be observed that there are several peaks which have abnormal amplitudes in the  $\pm$  and  $\mu$  frequency bands. These peaks may indicate a pathological case such as epilepsy, tumors, and traumas. In these three figures, the difference of the dominant frequencies can easily be seen. This means that by using Fourier transform, the frequency components that the signal includes can be estimated

Applications of wavelet transform and histograms of wavelet transform to some data can be seen in the figures 4,5,6,7,8 and 9.

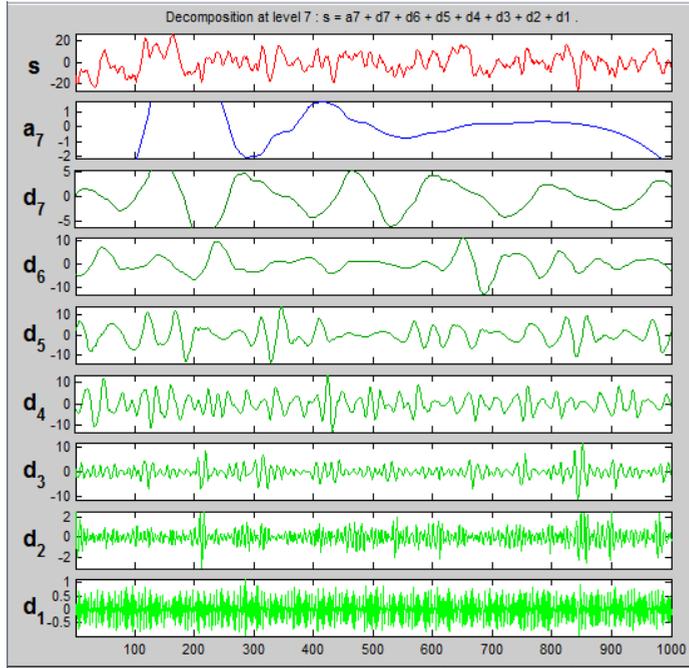


Figure 4- Application of db4 wavelet transform to EEG data of s1

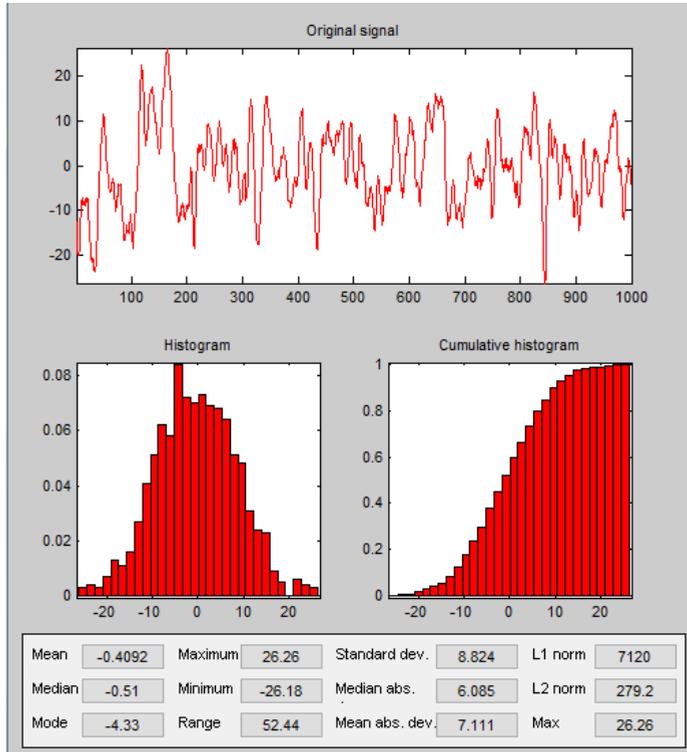


Figure 5- Histogram of db4 wavelet transform applied to EEG data of s1

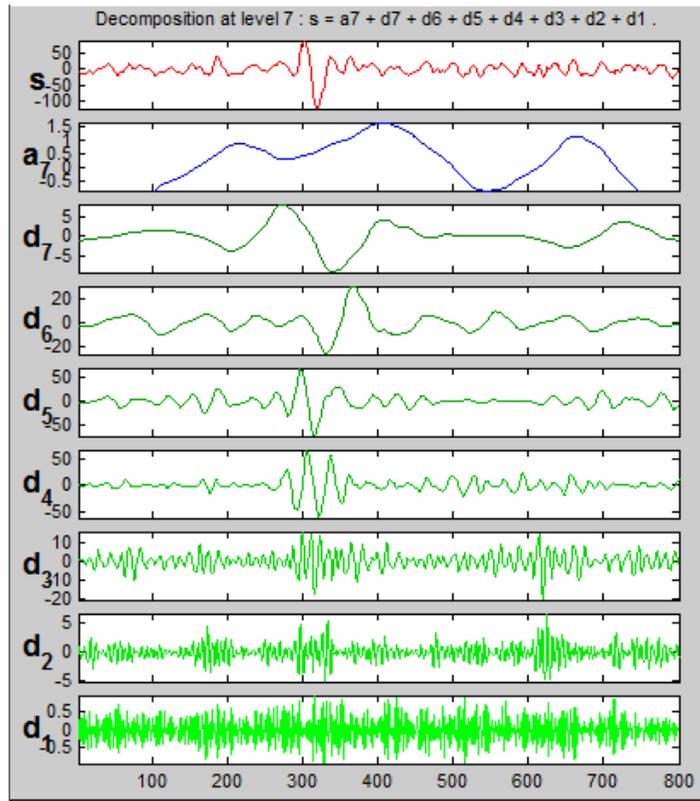


Figure 6- Application of db4 wavelet transform to EEG data of s2

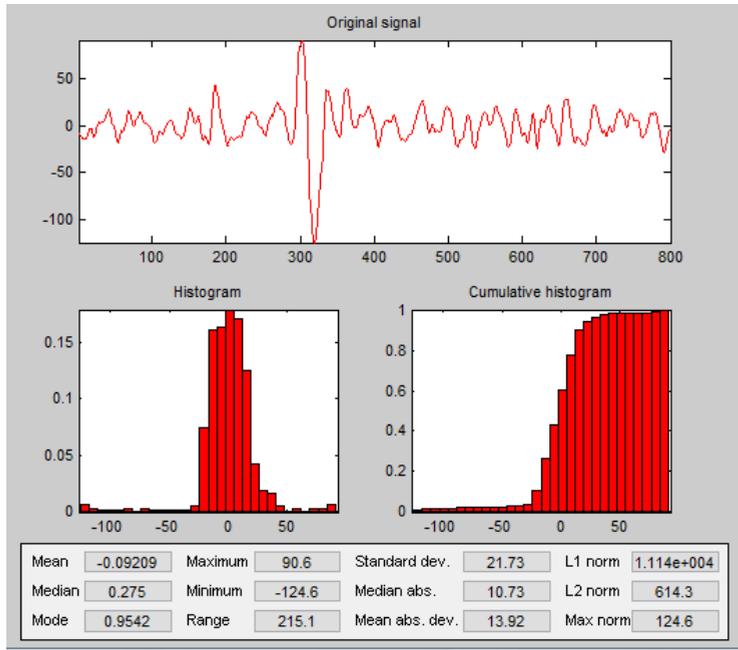


Figure 7- Histogram of db4 wavelet transform applied to EEG data of s2

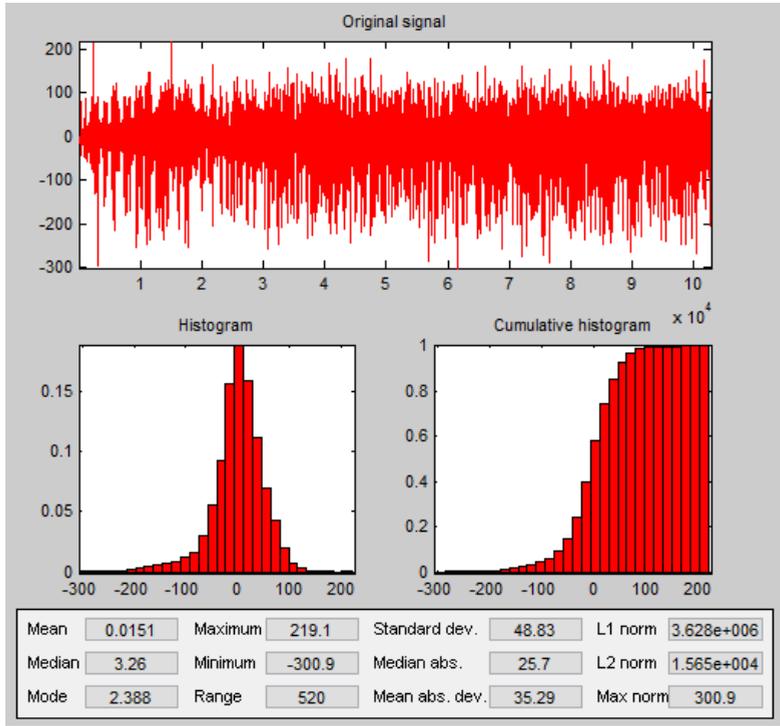


Figure 8- Application of db4 wavelet transform to EEG data of s3

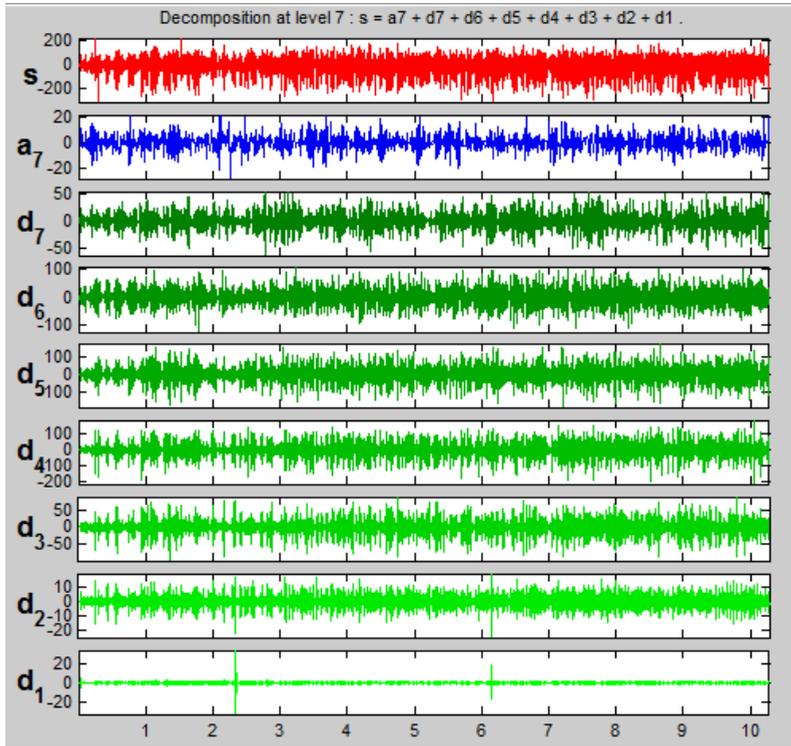


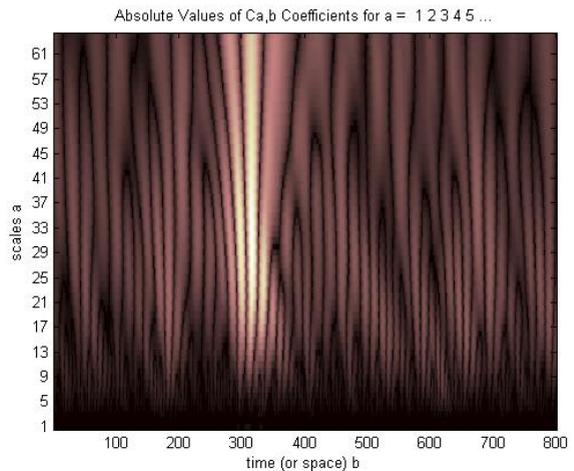
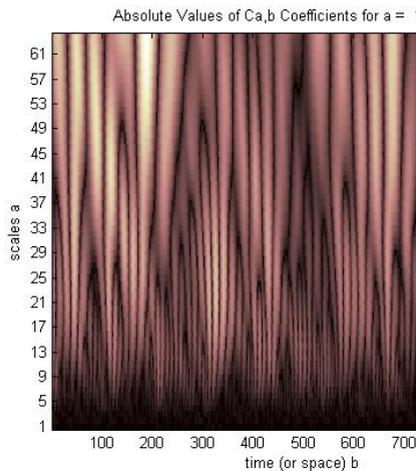
Figure 9- Histogram of db4 wavelet transform applied to EEG data of s3

In this section, relative wavelet energy coefficients corresponding to different band of frequencies of possibly epileptic patients are calculated and are compared to the relative energy distribution of one person who have no epilepsy and two persons who have epilepsy using MATLAB. For calculating relative energy 'db4' wavelet is used. Also two and three dimensional histograms are given related to three persons mentioned above.

	<b>d1</b>	<b>d2</b>	<b>d3</b>	<b>d4</b>	<b>d5</b>	<b>d6</b>	<b>d7</b>	<b>a7</b>
<b>S1</b>	0.0672	0.1789	1.8362	5.3261	7.6873	4.8232	4.4783	75.6027
<b>S2</b>	0.0211	0.3198	3.1970	50.5837	36.1856	9.9477	1.7802	17.9649
<b>S3</b>	0.0049	0.1459	4.0375	23.8916	34.3125	27.4314	8.7653	1.4108

Table 1- Relative energy distribution in non-epileptic EEG signals, in single seizure epileptic EEG signal and EEG signal having sequence of seizures

Above table shows that there is an energy transfer from approximate level to detail (the second line) which shows single pick. Also on analyzing the above table, the EEG signal having sequence of spike (the first line) i.e. the patient is suffering from epilepsy. Now most of the energy content is shifted to detailed levels and energy is considerably reduced in approximate level. These spikes in the figure 10 (Histogram 2D) and figure 11 (Histogram 3D) belong to two person's EEG signals



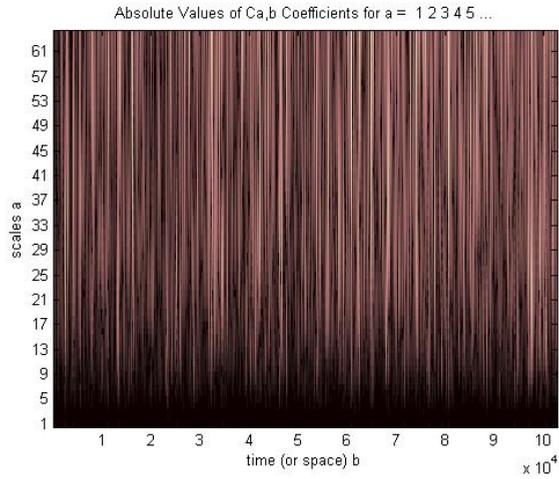
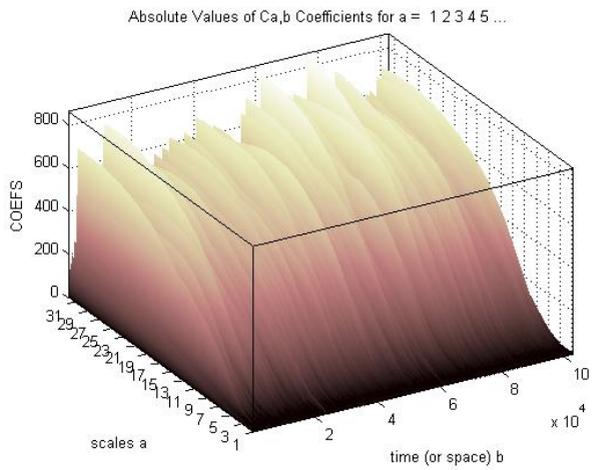
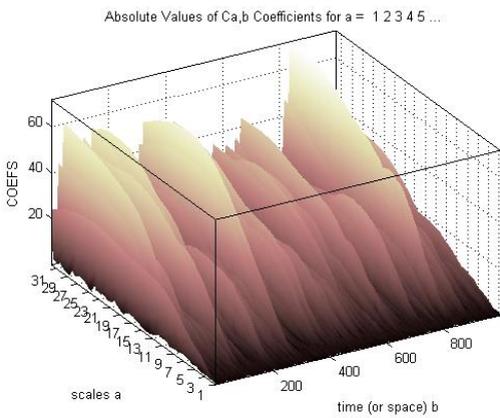


Figure 10- Comparison of 2D histogram for s1, s2 and s3



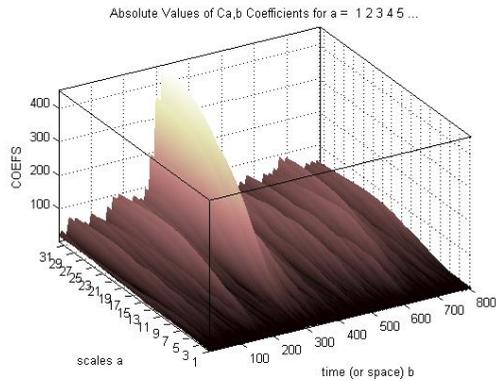


Figure 11- Comparison of 3D histogram for s1, s2 and s3

It is known that cross-correlation gives the similarity between three signals. Applying the concept of cross-correlation to s1,s2 and s3. It can be observed that the relative energy cross correlation of a sampled data s1,s2 and s3 representing normal, single spike and multiple spike respectively have different relative energy at approximate signal shown at instant 8 in the following figure. On the basis of approximate energy distribution we can predict the epileptic signal.

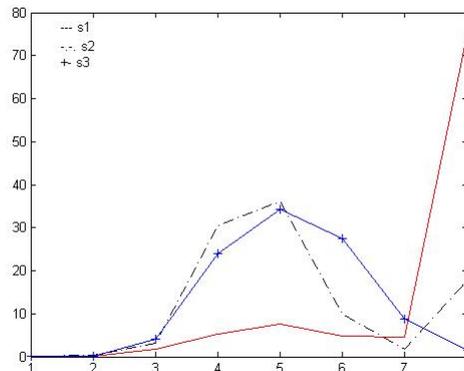


Figure 12- Comparison of energy distribution for s1, s2 and s3

## **5.CONCLUSION**

In this paper, both FFT and wavelet transform methods are applied to electroencephalogram (EEG) of possibly epilepsy patients. The signal on the basis of relative energy have been analyzed and are compared. Temporal and frequency variations of variables are detected by wavelet transforms. The results of this study are compared with previous studies. The comparison of results shows that the wavelet transform method is suited for detecting brain diseases. Beyond that, after analyzing all the data in 2 years, we may be able to develop a mechanism to predict future seizures for epileptic patients.

## 6. REFERENCES

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